An empirical model of Harold Jr.
Empirical IO problem set

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Consider the dynamic optimization problem of Harold Jr., who manages a larger fleet of buses and has to deal with a more volatile market than his father.

For each bus (i) in each month (t), Harold Jr. must decide whether to replace the bus's engine ($j_{it} = 1$) or perform regular maintenance ($j_{it} = 0$). He has the following current-period utility function:

$$u\left(j_{it}, x_{it}, \varepsilon_{it}\right) = \begin{cases} -\theta_1 x_{it} + \varepsilon_{0it} & \text{if } j_{it} = 0 \\ -\theta_2 rc_t + \varepsilon_{1it} & \text{if } j_{it} = 1 \end{cases}$$

where $x_t \in \{1, 2, \ldots, 7\}$ is the bus's mileage, rc_t is the price of a replacement bus engine, and the ε's are i.i.d. with a type-1 extreme value distribution.

Mileage evolves according to the following process:

$$x_{i,t+1} = \begin{cases} 1 & \text{if } j_{it} = 1 \\ \min\left(x_t + 1, 7\right) & \text{if } j_{it} = 0. \end{cases}$$

From Harold Jr.'s point of view, rc_t evolves exogenously according to the following process:

$$rc_t = \rho_0 + \rho_1 rc_{t-1} + e_t.$$

where e_t is normally distributed with standard deviation σ_ρ.

Harold Jr. has a discount factor $\beta = .95$ and acts to maximize expected discounted utility, $E\left[\sum_{t=1}^{\infty} \beta^t u\left(j_{it}, x_{it}, \varepsilon_{it}\right)\right]$. He has rational expectations.

1. Estimate $(\theta, \rho, \sigma_\rho)$ using a nested fixed point algorithm.
2. Estimate $(\theta, \rho, \sigma_\rho)$ without solving the value function in the estimation algorithm but relying on the Hotz-Miller inversion.
3. Estimate θ without solving the value function in the estimation algorithm and without any assumptions on how replacement costs evolve (i.e., estimate θ without using any estimate of (ρ, σ_ρ)).

The (simulated) data set includes decisions for 1000 buses observed over 100 months. The variables in the data should be self-explanatory: i, t, j, x, and rc.

Hints:

- You should discretize the state space for ρ to make solving the value function in 1 tractable, and to facilitate the computation of an integral based on the process for ρ in 1-2.
- For 2-3, you will need (smoothed) estimates of choice probabilities. For 2, you can estimate choice probabilities as a flexible function of rc and x. For 3, you should estimate choice probabilities for each t as a flexible function of x.
- Use whatever software you like. It might help to look at Rust's original code and data: http://gemini.econ.umd.edu/jrust/nfxp_description.html.

Please turn in the following via email to ptscott@gmail.com:

- A brief explanation of the steps involved in each estimation algorithm for 1-3, emphasizing the differences between each. Be clear about what objective function you are using.
- A table listing the point estimates for θ_1 and θ_2 from each estimation 1-3, and the length of time your algorithm for each estimation took.
- Your code.