Auctions

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Empirical IO

Winter 2015

Why study auctions?

- Non-trivial share of economic activity: auction houses, government procurement, oil leases, electricity, eBay, internet advertising
- Auctions sometimes have nice properties. E.g., second-price auction leads to efficient allocation without need for competition among sellers.
- Appealing for researchers:
  - The rules of the game and possible strategies are typically transparent (to agents and the econometrician)
  - Lots of data: usually, data on outcomes (winning price), sometimes also data on bids.

Types of auctions

- Number of items sold
  - Single-unit: oil leases, art sales, procurement contracts, timber
  - Multi-unit: treasury bills, spectrum, electricity
- Allocation and payment rule
  - In practice, can have sealed-bid and open outcry auctions. In theory, we think some of these procedures are isomorphic.
  - First-price auction $\approx$ Dutch auction $\approx$ descending-price auction
  - Second-price auction $\approx$ English auction $\approx$ ascending-price auction

Assumptions about bidders

- Private values: a bidder's valuation of the good is only a function of her own information; typically the bidder is fully aware of her own valuation.
- Common values: all bidders have the same valuation of the good, but each may only have an imperfect signal of the good.
- Interdependent/affiliated values: bidders don't have exactly the same valuations, but there is some correlation. E.g., another bidder's signal would be informative for me, but not as informative for me as my own signal. Such settings are much more difficult to analyze, theoretically or empirically.

Goals of empirical analysis

- Generally estimation aims at recovering bidders' valuations.
- Once bidder's valuations are known, we can infer welfare and predict how equilibrium would change if environment (e.g., auction rules) were changed.

Different approaches

- We will discuss three types of approaches:
  1. Explicit approach that solves for equilibrium (simulation)
     Laffont, Ossard, and Vuong (1995)
  2. Indirect approach based on necessary first-order conditions (more flexible)
     Guerre, Perrigne, and Vuong (2001)
  3. Alternative approach that does not rely on full model of optimal behavior (most flexible)
     Haile and Tamer (2003)

"Econometrics of First-Price Auctions"
Laffont, Ossard, and Vuong (1995)

Overview

- The auction: a descending (Dutch) auction for cases of eggplants (!!) Sellers: farmers. Buyers: resale firms/distributors.
- Observed: winning bid and some characteristics of the auction.
- Assumption: independent private values.
- Structural estimation recovers distribution of valuations. This could then be used to predict outcomes and welfare impacts of different auction designs, for instance.

Setup

- I symmetric bidders have valuations $v_i \sim F(\cdot|z_a, \theta)$.
- reserve price p_0.
- We would like to estimate F.
- Challenge: only winning price is observed.
- Strategy: use moments based on distribution of winning bid.

Equilibrium

- For now: ignore reserve price (see paper for details)
- Let $\beta(v)$ denote symmetric bidding function. For this to represent an equilibrium strategy:

$$\max_{b_i} (v_i - b_i) F\left(\beta^{-1}(b_i)\right)^{I-1}.$$

noting that $F\left(\beta^{-1}(b_i)\right)^{I-1}$ is probability of winning.

- First-order condition:

$$(v_i - b_i)(I-1) F\left(\beta^{-1}(b_i)\right)^{I-2} f(v_i) \frac{1}{\beta'(v_i)} - F\left(\beta^{-1}(b_i)\right)^{I-1} = 0$$

- Solution to this differential equation:

$$\beta(v_i) = e(v_i, I, F) \equiv v_i - \frac{\int_{\underline{v}}^{v} F(x)^{I-1} dx}{F(v_i)^{I-1}}$$

Winning bid

- What we observe is winning bid, or

$$\max_i \{\beta(v_i)\} = \max_i \{e(v_i, I, F)\} = e(v_{I:I}, I, F),$$

which is the bid of the individual with the highest value, noting that the $\beta(v)$ is a strictly increasing function.

- The winning bid satisfies

$$e(v_{I:I}, I, F) = \int_{\underline{v}}^{\infty} e(v, I, F)\, I \cdot F(v)^{I-1} f(v)\, dv$$

- For a given F, we could simulate the distribution of the winning bid, and then see how the simulated distribution matches the observed distribution. However, a simpler approach is possible based on the revenue equivalence theorem.

Revenue equivalence

Revenue Equivalence Theorem

Assume I risk-neutral bidders have independent private values drawn from common atomless distribution with full support on $[\underline{v}, \bar{v}]$. Any auction mechanism which is (i) efficient in awarding the object to the bidder with highest valuation and (ii) leaves any bidder with $v_i = \underline{v}$ with zero surplus yields the same expected revenue for the seller and results in the same expected payment for a bidder with a given valuation.

- This implies equivalence between first- and second-price auction, and the second-price auction is much easier to simulate (the winning bid is the second-highest valuation).

Estimation

- LOV assume a parametric class of distributions $F(\cdot|\theta, z_a)$
- For each θ and auction a,
  - draw I simulated valuations from $F(\cdot|\theta, z_a)$
  - sort draws and set $b_a^{w,s}$ and second highest valuation
  - With S simulations, approximate expectation of second-order statistic:

$$E(b^w; \theta, z_a) = S^{-1} \sum b_a^{w,s}$$

  - Note that $E(b^w; \theta, z_a)$ will be an estimate of the expected winning bid from the first or second-price auction due to revenue equivalence.
- Estimate θ with NLLS:

$$\min_{\theta} \sum_{a} \left(b_a^w - E(b^w; \theta, z_a)\right)^2$$

where b_a^w is the observed winning bid from auction a.

Egglplant auctions

- LOV observe winning bids and auction characteristics, and they assume that are distributed log-normally distributed around a linear function of the auction characteristics. i.e.,

$$E \log v_i = \theta' z_a$$

- We then add an iid normal shock to the above to get an individuals (log) valuation.
- They calibrate the variance of shocks with the variance of prices (can think of this as another moment). They don't actually observe number of bidders, so they do sensititivity analysis with respect to I.

TABLE I

Variables	Parameters: First Model	Parameters: Second Model
Number of Buyers (I)	11	18
Number of Simulations (S)	20	20
Number of Auctions (L)	81	81
Constant	0.1297 (0.02)	0.0286 (0.06)
Seller	−0.0107 (−0.17)	−0.0240 (−0.51)
Size 1	0.2402 (3.57)	0.2402 (4.39)
Size 2	0.1373 (1.39)	0.1213 (1.60)
Period	1.2404 (2.16)	1.1998 (2.90)
Date	0.3115 (3.04)	0.3202 (4.03)
Supply	−0.0340 (−0.59)	−0.0357 (−0.81)
Criterion Value	0.52395	0.51401

"Optimal Nonparametric Estimation of First-Price Auctions"
Guerre, Perrigne, and Vuong (2001)

Overview

- Main idea: write necessary first-order conditions for optimal bidding as a function of obsjects which can be recovered from data
- Can write FOC as a function of the distribution of bids (G) rather than the distribution of valuations (F)
- Monotonicity of bidding allows recover of distribution of valuations from distribution of bids – this can be done **non-parametrically**
- It's best if all bids are observed in practice, but estimation can still be done in principle with only observations of winning bid.

Equilibrium

- Recall bidder's problem:

$$\max_{b_i} \left(v_i - b_i\right) F\left(\beta^{-1}\left(b_i\right)\right)^{I-1}.$$

- First-order condition:

$$\left(v_i - b_i\right)\left(I-1\right) F\left(\beta^{-1}\left(b_i\right)\right)^{I-2} f\left(v_i\right)\frac{1}{\beta'\left(v_i\right)} - F\left(\beta^{-1}\left(b_i\right)\right)^{I-1} = 0$$

- Rearranging:

$$\beta'\left(v_i\right) = \left(v_i - \beta\left(v_i\right)\right)\left(I-1\right)\frac{f\left(v_i\right)}{F\left(v_i\right)}$$

Equilibrium

- First-order condition:

$$\beta'(v_i) = (v_i - \beta(v_i))(I-1)\frac{f(v_i)}{F(v_i)}$$

- Due to monotonicity, $G(b_i) = G(\beta(v_i)) = F(v_i)$, and

$$g(b_i) = f(v_i) \cdot 1/\beta'(v_i).$$

- Combining the above equations, we get a useful expression for equilibrium strategy:

$$v_i = b_i + \frac{G(b_i)}{(I-1)\,g(b_i)}$$

Estimation with observable bids

- We have derived an expression for an indvidual's valuation in terms of the distribution of bids:

$$v_i = b_i + \frac{G\left(b_i\right)}{\left(I-1\right) g\left(b_i\right)}$$

- This means we can look at the problem of recovering valuations as a problem of recovering the distribution of bids.
- When bids are observable, this is just a question of how well we can approximate the distribution. With tons of data on bids, it is trivial to construct a good non-parametric approximation to G (e.g., kernel functions).

Estimation with observable bids

- We can estimate g using kernels:

$$\hat{g}\left(b\right) = \left(A \cdot I\right)^{-1} \sum_{a} \sum_{i} \frac{1}{h} \mathcal{K}\left(\frac{b - b_{it}}{h}\right),$$

- and G using frequencies:

$$\hat{G}\left(b\right) = \left(A \cdot I\right)^{-1} \sum_{a} \sum_{i} 1\left(b_{ti} \leq b\right),$$

- and recover valuations:

$$\hat{v}_i = b_i + \frac{\hat{G}\left(b_i\right)}{\left(I - 1\right)\hat{g}\left(b_i\right)}$$

- Then we can similarly estimate f and F based on recovered $\hat{v}_i$.

Estimation with winning bids

- Sometimes we only observe winning bid. We can still use these same ideas and add one piece: the relationship between the distribution of the winning bid $G_{I:I}$ (first-order statistic) and the underlying bid distribution G.
- With independent private values, we get very simple relationship:

$$G_{I:I}(b) = G(b)^I$$

- In principle, this allows us to recover G from $G_{I:I}$. In practice, we may not observe many winning bids at low values. i.e., we only have substantial precision at the values of b which are observed as winning bids at a high probability.

"A Study of the Internal Organization of a Bidding Cartel"
John Asker (2010)

Overview

- A study of a bidding ring of stamp dealers, bidding on collectible stamps in New York auction houses.
- The ring used *knockout auctions*, internal auctions among members to allocate the good among ring members.
- The knockout mechanism leads to some interesting and counterintuitive effects:
  - Side-payments provided incentives to bid above valuations.
  - Overbidding sometimes caused inefficient allocations.
  - Overbidding sometimes increased the price received by sellers.
  - Overall, reduced competition more than compensated for the overbidding, and ring members benefited substantially from the scheme on average.

The Knockout Auctions

- Before the actual (target) auction, ring members could submit bids in knockout auction run by a hired agent.
- The ring's bidding limit in the target auction is the maximum price from the knockout auction. A bidding agent would submit the ring's bid.
- If the ring wins the target auction, the highest bidder from the knockout auction gets the item and may owe side-payments to other knockout participants.
  - "Sidepayments involve ring members sharing each increment between bids, provided that their bids are above the target auction price. Half the increment is kept by the winner of the knockout, and the balance is shared equally between those bidders who bid equal to or more than the "incremental" bid."

Example 1: Sidepayments from a successful acquisition in a target auction, Sotheby's, June 24, 1997, Lot 49

Knockout auction	Bid ($)	Sidepayment
Bidder A	9,000	$-\left(\frac{7{,}500-6{,}750}{2}\right) - \left(\frac{8{,}000-7{,}500}{2}\right) = -625$
Bidder G	8,000	$+\left(\frac{7{,}500-6{,}750}{2}\right) \times \frac{1}{2} + \left(\frac{8{,}000-7{,}500}{2}\right) = 437.50$
Bidder B	7,500	$+\left(\frac{7{,}500-6{,}750}{2}\right) \times \frac{1}{2} = 187.50$
Bidder J	5,100	0
Target auction price	6,750	

TABLE 2—BIDDING BY NUMBER OF BIDDERS IN THE KNOCKOUT

Number of Bidders	Target auction (winning bid)		Knockout auction (median bid)		% of lots won by ring	Total number of lots
	Mean	SD	Mean	SD		
1	733	1,262	616	1,134	19	623
2	1,314	2,016	1,066	2,048	36	367
3	2,014	3,246	1,750	3,029	48	260
4	2,217	3,492	2,293	4,082	69	196
5	2,249	3,419	2,092	3,322	68	144
6	2,098	2,628	2,163	3,014	74	91
7	2,979	3,425	3,655	4,116	86	74
8	4,790	4,904	6,233	7,726	96	26

Notes: Does not include the Harmer-Schau auctions. All subsequent analysis also excludes these auctions.

Bidder heterogeneity

TABLE 5—KNOCKOUT OUTCOMES, BY RING MEMBER

	All auctions ($n \geq 1$)		Auctions with at least 2 ring members interested ($n \geq 2$)			
Ring member	% high KO bid	# of knockouts	% high KO bid	% receive sidepayment	% pays sidepayments	# of knockouts
A	40	675	33	22	12	607
B	57	196	52	21	16	175
C	34	449	20	23	5	368
D	14	715	10	20	3	686
E	39	353	38	24	21	348
F	31	120	28	28	4	116
G	11	186	10	34	5	184
H	14	56	4	34	0	50
I	44	210	44	17	20	209
J	45	878	30	22	9	686
K	42	1,075	28	21	9	861

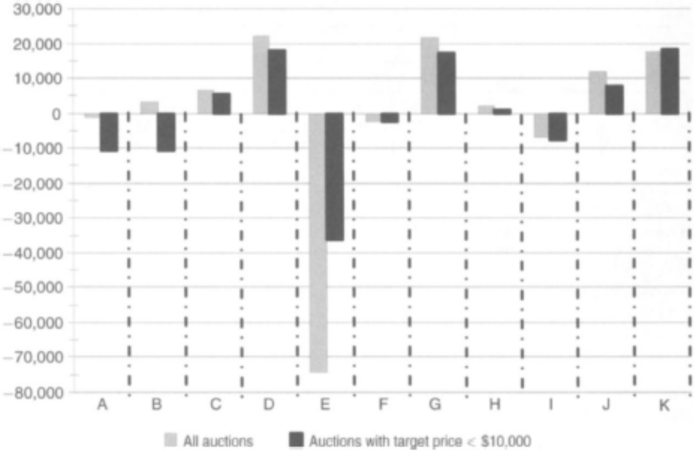

FIGURE 1. NET SIDEPAYMENTS FROM THE RING, BY MEMBERS IN DOLLARS

Bidder D: "My objective, basically was, you know, make money from these people as opposed to actually buying the stamps."

Naïve analysis

- Naïve estimates of damages can be easily calculated by assuming that knockout-auction bids represent true valuations.
- Then, the difference between the transaction prices in target auctions and the second highest bid in corresponding knockout auctions is a measure of damages (in cases where the second highest bid in the knockout was higher than the transaction price in the target auction).
    - Note: target auctions were English auctions.
- However, incentives created by sidepayments call for a more careful assessment.

TABLE 6—NAÏVE DAMAGES IN TARGET AUCTIONS WITH TWO OR MORE RING MEMBERS ACTIVE IN CORRESPONDING KNOCKOUT

By final price in target auction

	0–500	501–1,000	1,001–2,000	2,001–3,000	3,001–5,000	5,001–7,000	7,001–10,000	10,000+	Aggregate
Mean target price ($)	314	745	1,483	2,527	3,929	5,940	8,514	17,180	1,986
Mean highest knockout bid ($)	471	1,066	1,996	3,187	5,918	8,041	10,428	23,840	2,718
Mean total sidepayments ($)	42	92	154	245	622	697	526	1,910	222
Total naïve damages ($)	28,390	53,460	68,000	51,950	113,150	65,500	38,950	95,500	514,900
Mean naïve damages ($)	83	184	308	490	1,243	1,394	1,053	3,820	445
Number of lots won by ring	203	162	112	50	55	29	23	15	649
Total number of lots	341	290	221	106	91	47	37	25	1,158

By number of ring members in knockout

	2	3	4	5	6	7	8	Aggregate
Mean target price ($)	1,314	2,014	2,217	2,249	2,098	2,979	4,790	1,986
Mean highest knockout bid ($)	1,281	2,327	3,197	3,282	3,301	5,750	9,496	2,718
Mean total sidepayments ($)	12	96	249	211	365	895	1,898	222
Total naïve damages ($)	8,920	50,095	97,540	60,760	66,415	132,470	98,700	514,900
Mean naïve damages ($)	24	193	498	422	730	1,790	3,796	445
Number of lots won by ring	133	126	136	98	67	64	25	649
Total number of lots	367	260	196	144	91	74	26	1,158

Model basics

- Each bidder i has valuation in auction k of $v_{ik} \in [\underline{v}_i, \bar{v}_i]$ drawn from $F_i(v)$.
- Valuations are private and independently distributed, but not identically distributed across bidders.
- Ring members know the number of other bidders participating in a knockout, but not the identities.

Knockout bidding

- Expected profits:

$$\begin{aligned} \max_b \quad & \int_{-\infty}^{b} (v_{ik} - x)\, h_r(x)\, dx F_{-i}(\phi(b)) \\ & -\tfrac{1}{2} \int_{-\infty}^{b} \int_{x}^{b} (y - x)\, h_r(x)\, f_{-i}(\phi(y))\, dy dx \\ & -\tfrac{1}{2} \int_{-\infty}^{b} (b - x)\, h_r(x)\, dx \left(1 - F_{-i}(\phi(b))\right) \end{aligned}$$

  where

  - h_r is the density function for the highest nonring bid,
  - ϕ is the inverse strategy function,
  - α_j is the probability of j's participating in the auction,
  - and $F_{-i}(\phi(b)) = \left(\sum_{j \neq i} \alpha_j F_j(\phi_j(b))\right) / \sum_{j \neq i} \alpha_j$

Optimal bidding

- FOC for profit maximization:

$$\left(v_{ik}-b\right)h_{r}\left(b\right)F_{-i}\left(\phi\left(b\right)\right)+\int_{-\infty}^{b}\left(v_{ik}-x\right)h_{r}\left(x\right)dxf_{-i}\left(\phi\left(b\right)\right)$$

$$-\int_{-\infty}^{b}\left(b-x\right)h_{r}\left(x\right)dxf_{-i}\left(\phi\left(b\right)\right)+\frac{1}{2}\int_{-\infty}^{b}h_{r}\left(x\right)dx\left(1-F_{-i}\left(\phi\left(b\right)\right)\right)$$

Recovering valuations

- The first-order condition cannot be inverted for v in general, but with only two bidders,

$$v_{ik} = b - \frac{\frac{1}{2} H_r(b)\left(1 - G_{-i}(b)\right)}{\left(h_r(b)\, G_{-i}(b) + H_r(b)\, g_{-i}(b)\right)}$$

  where G_{-i} is the distribution function of b_{-i}.
- Asker focuses on auctions with two bidders to avoid identification issues.

Overbidding

- **Lemma 1** states that $\left.\frac{\partial \pi_{ik}}{\partial b_{ik}}\right|_{b_{ik}=v_{ik}} \geq 0$.
- Therefore, knockout bids are weakly greater than valuations.
- **Corollary**: the knockout auctions can lead to inefficient allocations.

Auction heterogeneity

- Extending the model to allow for unobserved auction-level heterogeneity, write valuations as:

$$u_{ik} = e^{x_k \gamma} \left(v_{ik} \varepsilon_k \right).$$

- Asker's structural approach recovers the distribution of v's and ε's. We're going to ignore details of dealing with the ε's here, but you should be able to see how the distribution of v's could be estimated if we don't have the ε's (think GPV).

Bidder heterogeneity

- For simplicity, he classifies bidders as either "weak" or "strong" and estimates a different distribution of valuations $F(\cdot)$ for each type.
- Remember that bidders don't know which other bidders are participating. Empirical frequencies of each bidder's participation are used for α_j's.

Notes on counterfactuals

- Solving for equilibria of the knockout auctions might be hard, but his counterfactuals are only English auctions, which are analogous to second price auctions and therefore easy to solve. This makes counterfactuals WAY easier.
- A difficulty is not knowing the distribution of (second highest) nonring bids.
  - U.B. assumption: second highest nonring value is equal to highest nonring valuation. This provides upper bound to damages. Why?
  - L.B. assumption: second highest nonring value is equal to minimum of highest nonring valuation and highest ring valuation. The provides lower bound to damages. Why?