Auctions I: Foundations

Lecture 5 (9.30am - 12.30pm)

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Empirical IO II - DEEQA
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Why do we study Auctions?

- Many markets are organized as auctions: art, government procurement, oil leases, electricity, treasury bills, eBay, etc.
- Auction as a price discovery mechanism to aggregate information.
- Advantages:
  - Rules of the market are clear and known by everyone (including the researcher)
  - Data availability: Actions (bids) and outcomes are oftentimes recorded.
- Possible to analyze the effects of entry, collusion, mergers, design changes (revenues, efficiency, prices, profits).

Auctions: Classification

- How many objects are sold (procured)?
  - Single unit: Oil leases, art sales, road constructions, timber tracts.
  - Multi unit: treasury bills, electricity, spectrum auctions.
- What's the mechanism by which goods are allocated and payments are computed?
  - (SU) First price: procurement, some timber tracts.
  - (SU) Second price: e-bay, art sales, internet advertising.
  - (MU) Discriminatory: some treasury bills.
  - (MU) Uniform: some treasury bills, electricity, spectrum auctions.

Auctions: Classification (cont)

- How is the bidding mechanism organized?
  - Sealed-bid: procurement, treasury bills, internet advertising, some timber tracts.
  - Open, Ascending: eBay, art sales, some timber tracts.
  - Open, Descending: some corporate debt securities and IPOs, some used car sales, some food markets.

Auctions: Classification (cont)

- How do bidders value the goods?
  - Private values: valuation only a function of own shock.
  - Common value: valuation is common, bidders receive a noisy independent signal.
  - Interdependent/affiliated values: some correlation, but also idiosyncratic component.

Empirical Analysis

- Usually interested in inferring fundamentals (bidder valuations) from observed bidding data (offers).
- Knowing about valuations allows to compute markups, $v - p$.
- Study how markups, rents, depend on forms of competition, properties of the good (private value vs common value), etc.
- Inferring fundamentals can also be useful to explore other issues related to the economic environment.

Empirical Analysis: Approaches

- Auction environments are well defined, strategic game understood.
- However, often theory helps little in characterizing solution to the auction (only for special cases).
- Theory is often more explicit about necessary first-order conditions.
- Two approaches:
  - Explicit approach that solves for the equilibrium outcome.
  - Indirect approach based on necessary first-order conditions.
- First approach needs to be parametric, second approach can be parametric or non-parametric.
- Alternative approaches do not impose full optimality conditions.

General References

- Ken Hendricks and Robert H. Porter, 2007. "An Empirical Perspective on Auctions," Handbook of Industrial Organization.
- Athey, Susan and Philip A. Haile, 2007. "Nonparametric Approaches to Auctions," Handbook of Econometrics.
- Paarsch, Harry J. and Han Hong, 2006. "An Introduction to the Structural Econometrics of Auction Data," MIT Press.

Today

- Laffont, Jean-Jacques, Herve Ossard and Quang Vuong, 1995. "Econometrics of First-Price Auctions," Econometrica, Econometric Society, vol. 63(4), pages 953-80.
- Guerre, Emmanuel, Isabelle Perrigne and Quang Vuong, 2000. "Optimal Nonparametric Estimation of FirstPrice Auctions," Econometrica vol. 68(3), pages 525-574.
- Philip A. Haile and Elie Tamer, 2003. "Inference with an Incomplete Model of English Auctions," Journal of Political Economy, University of Chicago Press, vol. 111(1), pages 1-51.

Simulation Approach

Laffont, Ossard and Vuong (1995)

- Structural estimation of first-price auction with independent private values (IPV).
- Auctioning method is descending (Dutch).
- Recover parametric distribution of bids.
- Parametric structural approach using simulated method of moments.
- Use it to infer issues of optimal design (auction format, reserve price).

Set-up

- I bidders, symmetric with IPV valuations $v_i \sim F(\cdot|z_I, \theta)$.
- Reserve price p_0.
- Goal is to estimate θ based on observed outcomes.
- Limitation: descending auction, only winning bid is observed.
- Construct moments for winning bids based on auction model.

Equilibrium Conditions

- Ignore reserve price for now (details in paper).
- Under symmetric strategies $\beta(v)$, bidder maximizes:

$$\max_{b_i} (v_i - b_i)F(\beta^{-1}(b_i))^{I-1}.$$

- First-order condition:

$$(v_i - \beta(v_i))(I-1)F(v_i)^{(I-2)}f(v_i)\frac{1}{\beta'(v_i)} - F(v_i)^{I-1} = 0.$$

- Differential equation, $b_i = \beta(v_i)$, with solution

$$e(v_i, I, p^0, F) = \frac{\int_{\underline{v}}^{v_i} xf(x)F(x)^{I-2}dx}{F(v_i)^{I-1}} = E[v_{I-1:I}|v_i = v_{I:I}].$$

Expected Winning bid

- Define winning bid as $b^w = e(v_{(I:I)}, I, p^0, F)$.
- Conditional on valuation being larger than p^0,

$$e(v_{I:I}, I, p^0, F) = \int_{p^0}^{\infty} e(v, I, p^0, F) I \cdot F(v)^{I-1} f(v) dv.$$

- One could simulate this object for a given distribution F.
- Simpler approach is possible.

Revenue Equivalence Theorem

Revenue Equivalence Theorem *Assume each of N risk-neutral bidders has a privately known signal X independently drawn from a common distribution T that is strictly increasing and atomless on its support* $[\underline{X}, \overline{X}]$. *Any auction mechanism which is (i) efficient in awarding the object to the bidder with highest signal; and (ii) leaves any bidder with the lowest signal* $\underline{X}$ *with zero surplus yields the same expected revenue for the seller, and results in a bidder with signal x making the same expected payment.*

- Equivalence between first-price and second price auction.
- Second-price auction winning bid much easier to simulate (second order statistic).

Estimation Steps

- For each parameter guess θ and each auction l,
  - Draw $v_1^s, \ldots, v_I^s$, simulated valuations from $F(\cdot|\theta, z_l)$.
  - Sort draws in ascending order.
  - Set $b_l^{w,s}$ as second highest valuation (or reservation price p^0).
  - Approximate expectation of second order statistic across simulations, $E(b_l^w;\theta) = \frac{1}{S}\sum_s b_l^{w,s}$.
- Estimate θ by NLLS:

$$\min_\theta \ \frac{1}{L}\sum_{l=1}^{L}(b_l^w - E(b_l^w;\theta))^2.$$

Empirical Specification

- Data from French eggplant market (81 auctions).
- Observe winning bid and auction characteristics (seller ID, case size, time control, total supply).
- Specify log valuation, log-normal assumption:

$$E \log v_l^i = \mu_l = \theta_1 + \theta_2 \text{ seller}_l + \theta_3 \text{ size } 1_l + \theta_4 \text{ size } 2_l + \theta_5 \text{ period}_l$$
$$+ \theta_6 \text{ date}_l + \theta_7 \text{ supply}_l \qquad (l = 1, \ldots, 81).$$

- Remaining challenges:
  - Calibrate variance of shocks with price variance.
  - Number of bidders not observed (sensitivity analysis).

Results

TABLE I

Variables	Parameters	
	First Model	Second Model
Number of Buyers (I)	11	18
Number of Simulations (S)	20	20
Number of Auctions (L)	81	81
Constant	0.1297	0.0286
	(0.02)	(0.06)
Seller	−0.0107	−0.0240
	(−0.17)	(−0.51)
Size 1	0.2402	0.2402
	(3.57)	(4.39)
Size 2	0.1373	0.1213
	(1.39)	(1.60)
Period	1.2404	1.1998
	(2.16)	(2.90)
Date	0.3115	0.3202
	(3.04)	(4.03)
Supply	−0.0340	−0.0357
	(−0.59)	(−0.81)
Criterion Value	0.52395	0.51401

FOC Approach

Guerre, Perringe and Vuong (2001)

- Main idea: re-arrange necessary first-order conditions as a functions of objects that are directly recoverable in the data.
- Transform FOC as function of distribution of bids (G), instead of valuations (F).
- Monotonicity of equilibrium bids with valuations allows to recover underlying valuation distribution.
- Distribution can be recovered non-parametrically.
- In practice, specially good if all bids are observed, but it is still applicable only if winning bid is observed (Athey and Haile, 2002).

Back to first-order conditions

- Equilibrium strategy given by,

$$\beta'(v_i) = (v_i - \beta(v_i))(I-1)\frac{f(v_i)}{F(v_i)}.$$

- Due to monotonicity, $G(b_i) = F(v_i)$,

$$g(b_i) = f(v_i) \cdot 1/\beta'(v_i).$$

- Can use expression to substitute equilibrium strategy,

$$v_i = b_i + \frac{G(b_i)}{(I-1)g(b_i)}.$$

Back to first-order conditions

- Powerful result: shift question to how well can we approximate bid distribution?
- Recovering valuations is then automatic under this framework, as everything is “observed.”
- Non-parametric identification as long as distribution of bids can be flexibly estimated.
- Typical approach: Kernel estimation based on observed bids.

Estimation Steps

- Approximate $\hat{G}(b)$ and $\hat{g}(b)$ from bidding data, e.g.,

$$\hat{g}(b) = \frac{1}{T \cdot I} \sum_t \sum_i \frac{1}{h} \mathcal{K}\left(\frac{b - b_{it}}{h}\right),$$

$$\hat{G}(b) = \frac{1}{T \cdot I} \sum_t \sum_i 1\left(b_{ti} \leq b\right).$$

- Recover valuations as

$$\hat{v}_i = b_i + \frac{\hat{G}(b_i)}{(I-1)\hat{g}(b_i)}.$$

- Fit density function using recovered sample of $\hat{v}_i$.

With only winning bids

- Relationship between winning bid and underlying distribution.
- Observing $G_{I:I}$ directly gives a representation of $G(b)$.
- Winning bid is first order statistic, CDF given by,

$$G_{I:I}(b) = G(b)^I.$$

- In practice, with many bidders, it might be hard to infer valuations at low ranges.
- Also general criticism for auctions estimation.
- First-order condition not be very accurately estimated if probability of winning is very very small.

Other extensions

- GPV has been very influential in the way auction data is analyzed.
- Many other models and derivations have been considered.
- Some examples:
  - Affiliated private values (Li, Perringe, Vuong, 2002).
  - Testing common values and private values (Haile, Hong, Shum, 2003).
  - Test RET (Athey, Levin and Seira, 2008).
  - Multi-unit auctions (Hortasu, 2002; Wolak, 2003).
  - Dynamics (Jofre-Bonet and Pesendorfer, 2003).
- ... and many more!

Minimal Assumptions

Haile and Tamer (2003)

- Context: US Forest Service timber auctions.
- Observe many auctions, number of bidders I and the K highest bids.
- Symmetric independent private values.
- Identification of English auctions (i.e. *ascending* auctions) under two simple assumptions:
  1. bidders never bid more than their valuations,
  2. bidders never let an opponent win with a bid below their valuation.

English auction and valuations

- In a second price auction, optimal to bid own valuation.
- It would be tempting to take $\hat{F}(v)$ as $\hat{G}(b)$.
- Revelation of bids on ascending auction does not happen for all bidders.
- Prices get revealed in jumps (when bidders quote a new price).
- Observed prices can jump, not all bidders might quote a price.

TABLE 2
Gaps Between First- and Second-Highest Bids

Quantiles	High Bid	Gap	Minimum Increment	Gap ÷ Increment
10%	9,151	30	4.1	1.2
25%	22,041	92	10.1	6.9
50%	55,623	309	23.4	14.8
75%	127,475	858	52.1	20.0
90%	292,846	2,048	110.5	76.4

Information in the Data I

- Data reveals distribution of K first order statistics.
- Estimate the probability that the i-th highest bid in all auctions with n bidders is below v non-parametrically.
- Possible to condition on number of bidders.
- Empirical likelihood:

$$\hat{G}_{i:I}(v) = \frac{1}{T_I} \sum_{t=1}^{T} 1\left(I_t = I, b_{i:I_t} \leq v\right).$$

Upper bound

- Bidders never bid more than their valuations:

$$b_{i:I} \leq v_{i:I} \implies F_{i:I}(v) \leq G_{i:I}(v).$$

- Fact:

$$\begin{aligned} F_{i:I}(v) &= \sum_{j=1}^{I} \binom{I}{j} F(v)^j (1 - F(v))^{I-j} \\ F(v) &= \phi(F_{i:I}(v), i, I) \end{aligned}$$

- This holds for any i and I. ϕ is monotone in its first argument.

$$F(v) \leq F_U(v) \equiv \min_{I \in [2,..,m], i \in [1,..,I]} \phi(G_{i:I}(v), i, I)$$

- Use $\hat{G}_{i:I}(v)$ to obtain an estimate.

Envelope

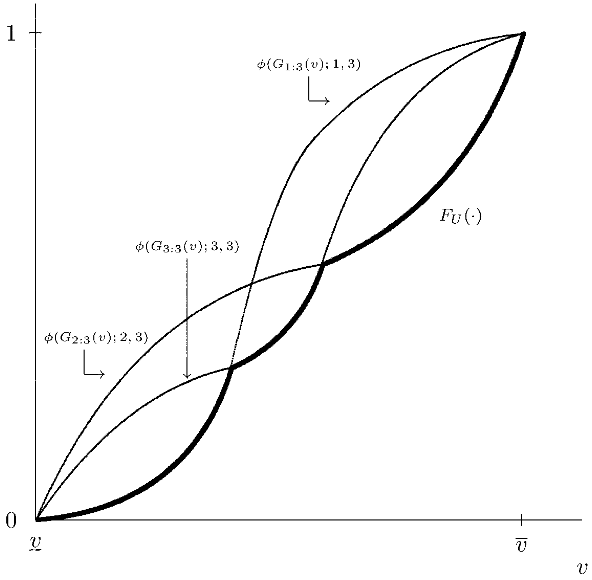

Fig. 1

Information in the Data II

- In ascending auctions, bidders can increase their bid if price is still below their valuation.
- Assume bidders do not let the price clear at their valuation v minus minimum increment (one penny).
- Winning bid plus increment will be above second highest valuation.
- Identify perturbed distribution (winning bid plus increment):

$$\hat{G}_{I:I}^{\Delta}(v) = \frac{1}{T_I} \sum_{t=1}^{T} 1\left(I_t = I, b_{I_t:I_t} + \Delta \le v\right).$$

Lower Bound

- Bidders do not allow an opponent to win if willing to beat:

$$v_{I-1:I} \leq b_{I:I} + \Delta \implies F_{I-1:I}\left(v\right) \geq G_{I:I}^{\Delta}\left(v\right).$$

- Fact:

$$\begin{aligned} F_{I-1:I}\left(v\right) &= F\left(v\right)^{I} + I \cdot F\left(v\right)^{I-1}\left(1 - F\left(v\right)\right) \\ F\left(v\right) &= \phi\left(F_{I-1:I}\left(v\right), I-1, I\right) \end{aligned}$$

- Because this holds for any I and ϕ is monotone in its first argument:

$$F\left(v\right) \geq F_{L}\left(v\right) \equiv \max_{I \in [2,..,m]} \phi\left(G_{I:I}^{\Delta}\left(v\right), I-1, I\right)$$

- Use $\hat{G}_{I:I}^{\Delta}(v)$ to obtain an estimate.

Putting bounds together

- Use observed bids to recover distribution of order statistics.
- Invert order statistic CDF to get bound on valuation CDF.

- Under the two assumptions,

$$\hat{F}_L(v) \equiv \max_{I \in [2,..,m]} \phi\left(\hat{G}^{\Delta}_{I:I}(v), I-1, I\right),$$

$$\hat{F}_U(v) \equiv \min_{I \in [2,..,m], i \in [1,..,I]} \phi\left(\hat{G}_{i:I}(v), i, I\right).$$

Inference

Theorem 3: *If $T_n/T \to \lambda_n \in (0,1)$ as $T \to 0$ for all $n \in \left\{2, 3..., \bar{M}\right\}$ then $\hat{F}_L(v) \to F_L(v)$ and $\hat{F}_U(v) \to F_U(v)$ a.s. and uniformly in v.*

- In practice, bounds may cross!
- Not too surprising, taking the minimum for the upper bound and the maximum for the lower bound.
- Use smoothing approach, replace min and max with weighted average:

$$\min(y_1, .., y_J) = \lim_{\rho \to -\infty} \sum y_j \left[\frac{\exp(y_j \rho)}{\sum_k \exp(y_k \rho)} \right]$$

and the max attains when $\rho \to \infty$.

Data

- Auctions for timber from US Forest Service.
- Focus on scaled sales where bidders pay for the quantities actually harvested (less scope for common values).
- Data on estimated value of particular forests, expected harvest, other plot details.
- IPV conditional on observables.

TABLE 3
Summary Statistics

	Mean	Standard Deviation	Minimum	Maximum
Number of bidders	5.7	3.0	2	12
Year	1985.2	2.6	1982	1990
Species concentration	.68	.23	.24	1.0
Manufacturing costs	190.3	43.0	56.7	286.5
Selling value	415.4	61.4	202.2	746.8
Harvesting cost	120.2	34.1	51.1	283.1
Six-month inventory*	1,364.4	376.5	286.4	2,084.3
Zone 2 dummy	.88		0	1

* In millions of board feet.

Results on Bounds

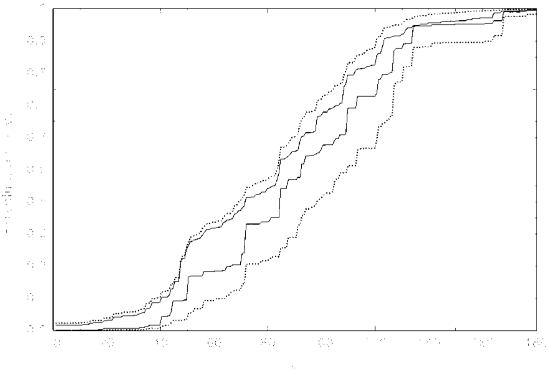

FIG. 10.—U.S. Forest Service timber auctions. Solid curves are estimated bounds, and dotted curves are bootstrap confidence bands.

Effects of observables

- Functional form assumption:

$$v_{it} = X_t \beta + \varepsilon_{it}$$

TABLE 5
FOREST SERVICE TIMBER AUCTIONS: SEMIPARAMETRIC MODEL OF BIDDER VALUATIONS (Modified Minimum Distance Estimates)

	Interval Estimate	95% Bootstrapped Confidence Interval
Constant	[8.8, 15.12]	[2.33, 18.15]
Species concentration	[13.19, 13.64]	[11.14, 16.54]
Manufacturing cost	[−.85, −.81]	[−1.02, −.79]
Selling value	[.61, .71]	[.57, .96]
Harvesting cost	[−.54, −.51]	[−.59, −.48]
Six-month inventory	[−.026, −.025]	[−.030,−.021]
Number of bidders	[.81, 1.23]	[.66, 1.24]

Bounds on Valuations and Optimal Reserve Price

- Valuations can be used to analyze further aspects of the auction.
- Optimal reserve price for an auctioneer with value v_0 (Myerson 1981; Riley and Samuelson 1981) maximizes,

$$\pi(p) = (p - v_0)(1 - F_0(p)).$$

- Bounds on F imply bounds in the function $\pi(p)$.
- Under the assumption that $\pi(p)$ is pseudo-concave, it also imply bounds on the optimal reserve price.
- Require objective function to be pseudo-concave.
- Requires reserve prices in the actual data to be low or zero compared to the optimal.

Bounds on Profits and Bounds on Optimal Reserve Price

- Profits bounded by,

$$\pi_1(p) = (p - v_0)\,(1 - F_U\,(p))\,,$$
$$\pi_2(p) = (p - v_0)\,(1 - F_L\,(p))\,.$$

- Let p_1^* maximize $\pi_1(p)$. Then,

$$p_L \equiv \sup\{p < p_1^* : \pi_2(p) \leq \pi_1^*\},$$
$$p_U \equiv \inf\{p > p_1^* : \pi_2(p) \geq \pi_1^*\}.$$

- Use empirical distributions to back these prices out.

Bounding Reserve Prices

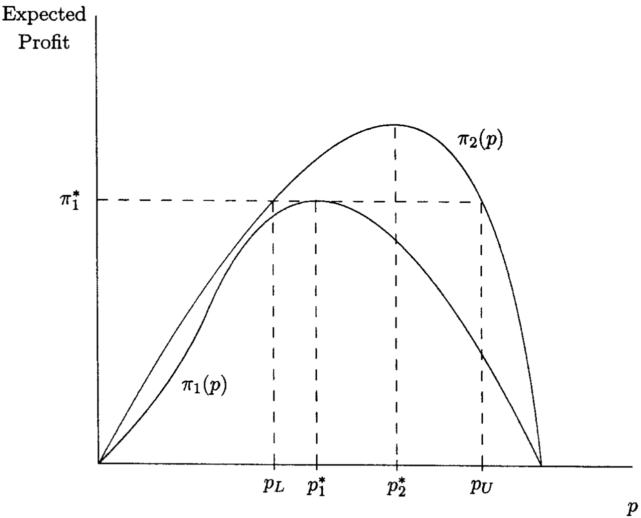

Fig. 2

Results on Optimal Reserve Prices

- They simulate outcomes under alternative reserve prices assuming $F = \hat{F}_{L|\bar{X}}(v)$ or $F = \hat{F}_{U|\bar{X}}(v)$.
- Compare with actual mean reserve price of $54 per MBF (area).
- Average gross revenue around $92.
- With $v_0 = 20$, gains from setting a close to optimal reserve price would be less than 10 percent (and not necessarily positive) even when $F = F_L$.
- With $v_0 = 80$, potential gains much larger.
- Suggests that Forest Service could improve revenues by putting more stringent reserve prices, and exploit high bids.

Bounding Reserve Prices

TABLE 4
SIMULATED OUTCOMES WITH ALTERNATIVE RESERVE PRICES

	Reserve Price					
	p_L		$(p_L + p_U)/2$		p_U	
	Distribution of Valuations					
	F_L	F_U	F_L	F_U	F_L	F_U
Reserve price when $v_0 = \$20$	62.40		86.02		109.65	
Change in profit	6.96	−2.78	6.67	−2.74	1.74	−18.57
Pr(no bids)	.00	.02	.07	.12	.19	.41
Reserve price when $v_0 = \$40$	74.93		92.29		109.65	
Change in profit	7.64	−.61	7.61	−1.14	6.30	−10.04
Pr(no bids)	.03	.05	.11	.18	.19	.41
Reserve price when $v_0 = \$60$	85.67		103.39		121.11	
Change in profit	9.23	1.92	12.04	3.14	7.21	−6.05
Pr(no bids)	.07	.12	.15	.28	.35	.58
Reserve price when $v_0 = \$80$	98.20		112.34		126.48	
Change in profit	13.65	7.63	15.03	6.82	10.44	.96
Pr(no bids)	.13	.24	.28	.46	.46	.72
Reserve price when $v_0 = \$100$	111.09		122.54		134.00	
Change in profit	20.09	15.94	21.65	16.87	17.00	14.30
Pr(no bids)	.28	.45	.45	.60	.67	.80
Reserve price when $v_0 = \$120$	144.74		156.01		167.29	
Change in profit	32.06	31.31	33.72	31.64	31.56	28.87
Pr(no bids)	.84	.86	.84	.89	.88	.97

NOTE.—Profit and reserve price figures are given in 1983 dollars per MBF. See text for additional details.

Next week

- Interdependent Costs (Somaini, 2014)
- Dynamic Auctions (Jofre-Bonet and Pessendorfer, 2003)
- Multi-Unit Auctions (McAdams and Hortasu, 2010)