Auctions II: Extensions

Lecture 6 (9.30am - 12.30pm)

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Empirical IO II - DEEQA
Toulouse School of Economics

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Last week re-cap

- We looked at identification of valuations under three different settings:
  - Only winning bid of descending auction is observed (first price).
  - All bids of sealed-bid are observed (first price).
  - Bids are observed in ascending order, but they are not necessarily complete (second price).
- For all papers, we considered the case of symmetric independent private values (SIPV).

Relaxing the IPV assumption - Campo, Perrigne and Vuong (2003)

- Several papers have worked on extending the methods of GPV and HT to in a more general setting.
- Campo, Perrigne and Vuong (2003) extend GPV to Asymmetric Affiliated Private Values.
- Affiliation: probability distribution across agents is not the product of distributions, correlation.
- Equation same as before, but now need modified bid distributions that account for correlation.
- Condition distributions on own bid (double kernel on others bids and own bid).

Unobserved Heterogeneity - Krasnokutskaya (2011)

- Assume conditional independent private values (CIPV).
- Remaining correlation in bids due to unobserved heterogeneity (observed to bidders and auctioneer).
- Use deconvolution methods and characteristic functions to identify *distribution* of unobserved heterogeneity.
- Can test against independent private values and unobserved heterogeneity (against remaining affiliated private values).
- Highway procurement: reject IPV, but not UH.
- Athey, Levin and Seira (2011) find unobserved heterogeneity is relevant in timber auctions.
- Roberts (2008) presents alternative approach using reserve price (under monotonicity).

Correlation in English Auctions - Aradillas-Lopez, Gandhi and Quint (2013)

- Aradillas-Lopez, Gandhi and Quint (2013) extend HT to Correlated Private Values (still symmetric).
- Allow positive correlation: if many bidders are below a certain value v, then other bidder also more likely below.
- Assume transaction price above reserve price and second highest bid (no Δ).
- In timber auctions, show that optimal reserve prices might be wildly overestimated if correlation is ignored.
- Strategic reserve price increases revenues when $v_{I-1:I} \leq r \leq v_{I:I}$, but decreases payoffs if $v_0 \leq v_{I:I} \leq r$.
- Positive correlated valuations reduce the probability of the first event and increases the probability of the second event.

Today

- Somaini, 2011. "Competition and Interdependent Costs in Highway Procurement," Working Paper, MIT.
- Mireia Jofre-Bonet and Martin Pesendorfer, 2003. "Estimation of a Dynamic Auction Game," Econometrica, Econometric Society, vol. 71(5), pages 1443-1489, 09.
- Ali Hortasu and David McAdams, 2010. "Mechanism Choice and Strategic Bidding in Divisible Good Auctions: An Empirical Analysis of the Turkish Treasury Auction Market," Journal of Political Economy, University of Chicago Press, vol. 118(5), pages 833-865.

Interdependent Costs

Somaini (2011)

- Consider effects of interdependent valuations on competition in a setting with subsidies to small firms.
- Interdependent model with common cost component, winner's curse.
- Bidder with best valuation might refrain from bidding
- Subsidized bidders might gain even more often.
- Goal: explore identification of signals/common value component and assess effects of competition.
- Key: use predetermined cost shifters that are observable.

Set-up

- Model with:
  - asymmetric bidders,
  - nonindependent private information,
  - and interdependent costs (common component).
- Asymmetries are not a nuisance, but generate the variation needed to identify the model.

General Framework - Notation

- The auctioneer procures the completion of a project, and runs a first-price auction between N risk-neutral bidders.
- C_i completion cost to firm i. Random variable that is realized after auction.
- S_i: Firm i's signal. $S = \{S_1, S_2, ..., S_N\}$ and $S_{-i} = S \backslash S_i$. E.g., equipment and labor capacity constraints, expectation about future inputs markets. Bidder i knows his own s_i.
- B_i: Bid of bidder i
- d_i: publicly observable characteristic of bidder i.
- w_0: Other observed project characteristics (not important, drop in notation in slides).
- w: All public info (d, w_0).

Information and expected costs

- Expected cost of firm i at the time of the auction:

$$E\left(C_i | s_i, d\right).$$

- Full information expected cost function (all signals observed):

$$E\left(C_i | s_i, s_{-i}, d\right).$$

- Private costs Hypothesis: competitors' signals do not affect the costs forecasts.

$$E\left(C_i | s_i, s_{-i}, d\right) = E\left(C_i | s_i, d\right)$$

- Model fundamentals that are identified:

$$\left\{F_{S|D}, E\left(C_i | S, D\right)\right\}_{i=1}^{n}.$$

Assumptions

1. Firms are risk neutral.
2. Signals are one-dimensional random variables distributed as uniform $[0, 1]$. The joint density is continuous and bounded.
3. Cost shifters and signals are independent: $F_{S|D} = F_S$.
4. Exclusion Restriction: $E\left(C_i|s_i, s_{-i}, d\right) = E\left(C_i|s_i, s_{-i}, d_i\right)$ which is continuous in s, d and strictly increasing in d_i and s_i
5. The data are generated by a unique Bayes Nash Equilibrium.
6. Each bidder strategy $\beta_i\left(s_i, d\right)$ is a monotone function of s_i:

$$s_i = P\left(B_i < b_i|d\right) = G_{B_i}\left(b_i|d\right)$$

What can we learn from the data?

- What is a *identified* with the above assumptions?
  - Joint distribution of signals and costs $F_{S,C|d}$ not identified.
  - Joint distribution of signals and full information expected cost functions: $\left\{F_S(.), \{E(C_i|s_i, s_{-i}, d_i)\}_{i=1}^N\right\}$
- Enough to compute most counterfactuals as long as additional information is not revealed at an intermediate step.
- Identification of $F_S(.)$: Recall $s_i = G_{B_i|d}(b_i)$, then it is possible to obtain the joint distribution of signals:

$$F_S(s) = G_{B|d}\left(G_{B_1|d}^{-1}(s_1), \ldots, G_{B_N|d}^{-1}(s_N)\right)$$

Identification of the full information cost

- Firm i is best-responding to its competitors' strategies.
- Let $M_i = \min_{j \neq i} \beta_j (S_j, d)$.
- The expected residual demand (Pr. Win) of bidder i is:

$$P(M_i > bid | s_i, d) = 1 - G_{M_i|B_i,d}(bid|b_i)$$

- First order condition of i's optimization problem:

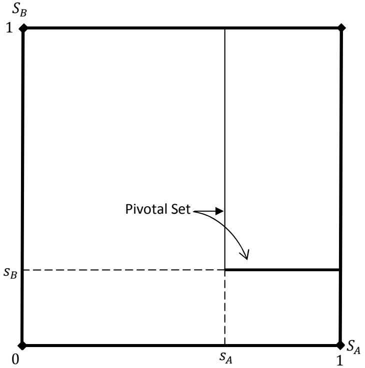
$$\begin{aligned} \text{Exp Mg Cost} &= \text{Exp Mg Revenue} \\ E[C_i|s_i, M_i = b_i, d_i] &= b_i - \frac{1 - G_{M_i|B_i,d}(b_i|b_i)}{g_{M_i|B_i,d}(b_i|b_i)} \end{aligned}$$

- The (expected) marginal revenue is identified from the data.
- The (expected) marginal cost is the expected cost conditional on s_i, d_i and the event "bid b_i is pivotal".

Role of being pivotal

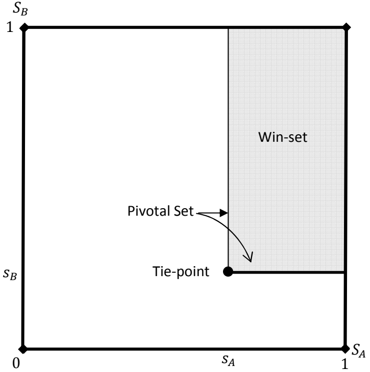
- If costs are correlated, additional information if bidder conditions on setting the price.
- Correction for the winner's curse.
- First-order condition chosen at the margin, when bidder sets the price.
- Need to identify pivotal set in the data to backup expected markup at the margin.
- Identify distribution of bids when firm just ties with others, conditional on its own bid.
- Need substantial variation in d to explore different regions.

Identification of full information costs



Suppose firm i ties at s_A and s_B for given d.

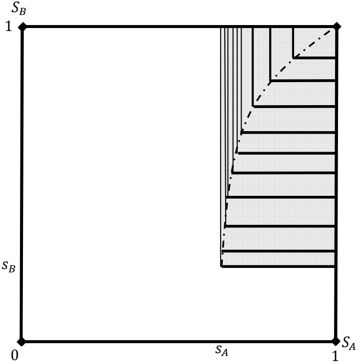
Identification of full information costs



Suppose firm i ties at s_A and s_B for given d.

s_A and s_B will be different for different d_{-i}.

Identification of full information costs



Suppose firm i ties at s_A and s_B for given d.

s_A and s_B will be different for different d_{-i}.

Intuition: Use variation in d_{-i} to find a different "pivotal set" holding constant (s_i, d_i).

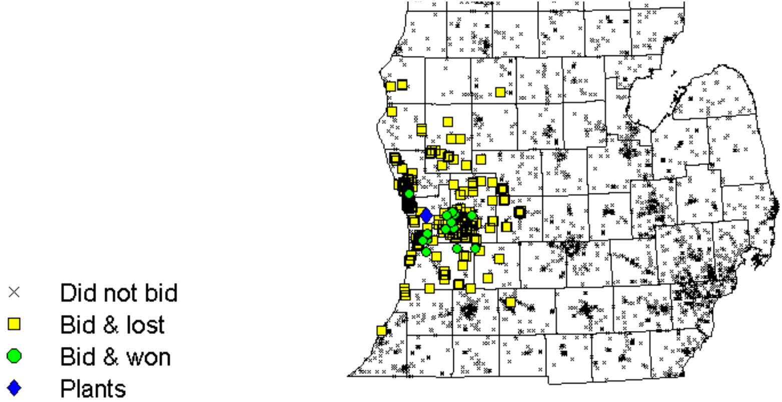
Application

- Hot-Mix-Asphalt in Michigan procurement auctions.
- Plant production: mix asphalt with aggregates at about 300° F (150°C).
- Paving and compaction must be performed while the asphalt is sufficiently hot.
- Distance is an important determinant of costs. Firms need to transport the HMA from their plants to the project site.
- Plant locations are predetermined.
- Project locations introduce variation in the vector of distances from plants to projects.
- Firms also compete in the input and subcontracting markets.

Practical issues

- Practical implementation will need to deal with the curse of dimensionality.
- In theory, non-parametric. In practice, rely on flexible functional forms.
- Use engineering estimates for projects to calculate standardized bids and simplify observable auction heterogeneity.
- Variation in d is driven by participation, need to properly account for entry with censoring.
- Need semi-parametric form to potentially extrapolate to regions where no entry is observed.
- Use latitude and longitude instead of distance of each firm.

Distance, Participation and Winning Probability



Data

Table 1(a): Descriptive Statistics. Engineer's estimate, bids and distances.

Variable	N	Mean	Sd	P5	Median	P95
Engineer's estimate ($000)	3,851	1,398	3,117	125	656	4,477
Lowest bid ($000)	3,851	1,320	2,983	118	603	4,236
Participants	3,851	5.08	3.45	2	4	12
(2nd Lowest/Lowest bid-1)×100	3,770	6.9	7.7	.4	4.7	20.9
(Lowest/engineer-1)×100	3,851	-6.4	12.6	-25.6	-7	14.5
Distance of Winner (km)	3,662	40	48	2	27	122
Distance of Bidder (km)	18,778	51	50	4	38	138

Note: Pct stands for percentile. 2nd Lowest: the second lowest bid. engineer: engineer's estimate. In 81 auctions there was only one bid. There were 189 auctions won by a firm for which I did not find any verifiable location.

Estimation: Parameters of the model

- Any fully nonparametric estimation would be plagued by the curse of dimensionality.
- I assume some functional forms:
  - $F_S(.)$ is assumed to be a Gaussian copula with covariance matrix Σ. Moreover, Σ is assumed to have a factor structure: $LL' + \Lambda$, where L is a N-by-l loading matrix and Λ is a diagonal matrix.
  - Full information costs are assumed to be additively separable in auction covariates and competitors signals.

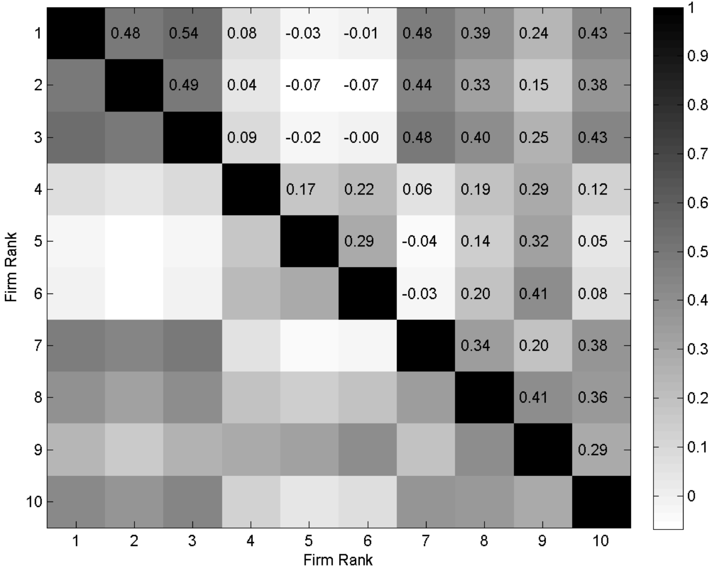
$$E\left(C_i|s, d_i, w_0\right) = \delta_{wi}\left(s_i\right) w_0 + \delta_{di}\left(s_i\right) d_i + \sum_{j \neq i} \delta_{ji}\left(s_i\right) \psi\left(s_j\right) + \delta_{ii}\left(s_i\right)$$

  - The identification argument required n-dimensional variation in cost shifters. I exploit variation in project location which is essentially 2 -dimensional: $\delta_{ji}\left(s_i\right) = \delta_{ki}\left(s_i\right)$.
- Parameters of the model: L and δ.

Estimation of the joint distribution of signals

- Estimation of the marginal distribution of bids:
  $s_i = G_{B_i|d,w_0}(b)$
  - Dimensionality of d:
    - I replace (d, w_0) by $\tilde{w} = (latitude, longitude, w_0)$
    - I estimate the probability of entry, the expected bid and variance of bid semi-parametrically. I use a 10km-bandwidth Gaussian Kernel for $(latitude, longitude)$.
    - Obtain $\hat{s}_i = \hat{G}_{B_i|\tilde{w}}(b)$, where non-participation implies $b_i = \infty$
- Estimation of the joint distribution of signals:
  - Censoring: I can only recover $\hat{s}_i = \hat{G}_{B_i|\tilde{w}}(b)$ when a firm submitted a bid.
    - Bidders first observe their signals and then they decide to participate or not. Therefore, if the firm does not participate I can only infer that $s_i >$ probability of participation.
    - The likelihood of an censored signal can be written in terms of the parameters L (Λ is restricted so that $LL' + \Lambda$ has ones in the main diagonal).
    - I estimate the parameters L by simulated maximum likelihood. (Tobit Factor Model, Kamakura & Wedel 2001).

Signals


0.48
0.54
0.08
-0.03
-0.01
0.48
0.39
0.24
0.43
0.49
0.04
-0.07
-0.07
0.44
0.33
0.15
0.38
0.09
-0.02
-0.00
0.48
0.40
0.25
0.43
0.17
0.22
0.06
0.19
0.29
0.12
0.29
-0.04
0.14
0.32
0.05
-0.03
0.20
0.41
0.08
0.34
0.20
0.38
0.41
0.36
0.29
1
2
3
4
5
6
7
8
9
10
Firm Rank
Firm Rank
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

Full information costs

- Recall:

$$E\left(C_i|s, d_i, w_0\right) = \delta_{wi}\left(s_i\right) w_0 + \delta_{di}\left(s_i\right) d_i + \delta_{ji}\left(s_i\right) \sum_{j \neq i} \psi\left(s_j\right) + \delta_{ii}\left(s_i\right)$$

- The marginal cost is the expected cost conditional on s_i, d_i and the event "bidder i is pivotal": $M_i = b_i$.

$$mc_i = \delta_{wi}\left(s_i\right) w_0 + \delta_{di}\left(s_i\right) d_i + \delta_{ji}\left(s_i\right) \sum_{j \neq i} \tilde{\psi}_j\left(b_i, d\right) + \delta_{ii}\left(s_i\right)$$

  where $\tilde{\psi}_j\left(b_i, d\right) = E\left[\psi\left(S_j\right)|M_i = b_i, d, b_i\right]$.

- Estimate of the marginal cost:

$$m\hat{c}_i = b_i - \frac{1 - \hat{G}_{M_i|B_i,d}\left(b_i|b_i\right)}{\hat{g}_{M_i|B_i,d}\left(b_i|b_i\right)}$$

- Estimate of $\tilde{\psi}_j\left(b_i, d\right)$ can be obtained by numeric integration.

Testing and Estimation.

- Recall,

$$m\hat{c}_i - \delta_{ji}(s_i) \sum_{j \neq i} \hat{\psi}_j(b_i, d) = \delta_{wi}(s_i) w_0 + \delta_{di}(s_i) d_i + \delta_{ii}(s_i)$$

- Under the true $\delta_{ji}(s_i)$, the expression on the right should not depend on d_{-i}.
- Testing the private cost hypothesis: Does the distribution of $m\hat{c}_i$ depend on d_{-i}?
- Estimation: Chernozhukov and Hansen (2005) IVQR: find $\delta_{ji}(s_i)$ to minimize the Wald statistic on the coefficients on competitors' distance d_{-i}.
- Buchinsky and Hahn (1998): "An Alternative Estimator for the Censored Regression Quantile".
  - Intuition: the τ-th quantile of the uncensored distribution is that $\frac{\tau}{\pi(\tilde{w})}$-th quantile of the censored distribution.

Estimation Results - Summary

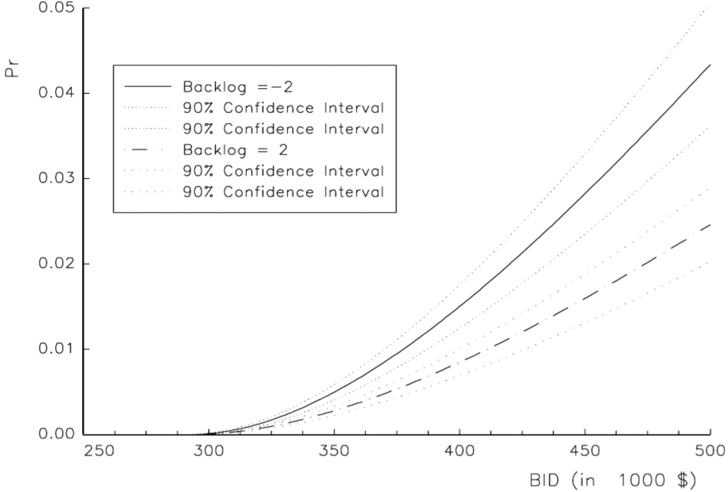
Firm	Effect of signals	
	Own	Competitors
1	0.100***	0.027***
2	0.131***	0.029***
3	0.075***	0.017***
4	0.115***	0.014***
5	0.143***	0.008 **
6	0.025 **	0.007 **
7	0.115***	0.015 **
8	0.138***	0.014 *
9	0.183***	0.011
10	0.087***	0.014***

Dynamics

Jofre-Bonet and Pesendorfer (2003)

- Consider the effect of dynamic factors on optimal bidding.
- In particular, capacity constraints and backlog in procurement contracts.
- Empirical observation: winning a contract before reduces probability of winning again.
- Similar approach to GPV, but adapted dynamic first-order conditions.
    1. First, estimate distribution of bids conditional on state variables (parametric).
    2. Second, reconstruct FOC to estimate costs, accounting for value function.
- Parallels dynamic estimation.

Motivation



Pr
0.05
0.04
0.03
0.02
0.01
0.00
Backlog =−2
90% Confidence Interval
90% Confidence Interval
Backlog = 2
90% Confidence Interval
90% Confidence Interval
250
300
350
400
450
500
BID (in 1000 $)

Set-up

- Consider two sets of bidders: regular (dynamic) and fringe (non-dynamic).
- Regular bidders have capacity constraints (backlog).
- Fringe bidders always bid, regular bidders might have opportunity cost larger than reserve.
- State characterized by size of current projects for each regular bidder (s_i).
- Transition of contract size is deterministic: depleted linearly based on planned completion, enlarged upon winning a contract.
- Bidders get marginal cost draws, c_i, and common draw on contract characteristics, η.

Problem of the firm

- Given believes on what other firms will do:

$$V_i(s) = E\Big[\max_b \, (b - c_i) Pr(i \text{ wins}|b, \eta, s) + \\ + \beta \sum_j Pr(j \text{ wins}|b, \eta, s) V_i(\omega(\eta, s, j)) \Big],$$

  with ω giving the transition function for s if a firm j wins and V_i represents the *expected* NPV (expectation over s_0 and c_i).

- First-order condition implies:

$$b = c_i + \frac{1}{\sum_{j \neq i} h(b|\eta, s_j, s_{-j})} \\ + \beta \sum_{j \neq i} \frac{h(b|\eta, s_j, s_{-j})}{\sum_{l \neq i} h(b|\eta, s_l, s_{-l})} [V_i(\omega(s_0, s, j)) - V_i(\omega(s_0, s, i))].$$

Aside on $h(.)$

- $h(.)$ is the generalized analog term in GPV:

$$h(.|\eta, s_i, s_{-i}) = \frac{g(.|\eta, s_i, s_{-i})}{1 - G(.|\eta, s_i, s_{-i})}.$$

- State dependent, harder to estimate in practice (parametric).
- Note: This is a procurement auction.
- Higher bid implies *lower* probability of winning.
- $1 - G(.)$ instead of $G(.)$ in the equation.

Estimation Strategy

- Express V_i as function of transitions and payoffs:

$$V_i = [I - \beta B_i]^{-1} A_i,$$

with B_i as transition matrix, A_i as expected payoff vector.
- Key: Write A_i and B_i as a function of distribution of bids.
- B_i estimated from probabilities of winning and exogenous process on η.
- For A_i, substitute equilibrium from FOC into expected value function equation to achieve expression that only depends on distribution of bids.
- Once V_i is estimated, can recover distribution of c_i in FOC.

Application

- Highway and street construction procurement auctions in California.
- Information on bids from 1996 to 1999.
- Contract characteristics: date, location, reservation price, planned time and engineering cost estimate.
- Create measure of capacity also from past data on actual contracts.
- Focus on the effect of backlog: current running contracts.
- Reduced form suggests that backlog affects negatively on bidding participation and inflates bids.

Results

- Hazard function of bid distribution is conditional on s and η.
- Non-parametric approach would be hard.
- Authors use some previous theoretical work to motivate a Weibull distribution as a function of observable contract characteristics and backlog.
- Estimate using likelihood function, need to make sure it is well defined (parameter restrictions).
- Consider specification with bidder-specific backlog effects.
- Confirm backlog as an important bid shifter.
- Also sum of backlog predictive of bids (positive effect).

Cost distribution

- Given $\hat{g}$, $\hat{G}$, they recover value function and costs.
- NPV is decreasing as a function of current backlog.
- They also find that implied cost distribution is state-dependent.
- Backlog can have general equilibrium implications to input costs.
- Balat (2013) explores these forces in the context of the stimulus package.

Multi-Unit Auctions

Hortasu and McAdams (2010)

- Revenue equivalence (RET) between first and second price auction under standard assumptions, is a well-know result for single unit auctions.
- Stylized alternative models of multi-unit auctions are uniform-price auction and discriminatory auction.
- Result does not hold anymore, even under same strict assumptions.
- Goal: assess empirically whether a discriminatory price auction yielded higher revenues than a uniform-price auction.
- Non-parametric set identification (step bids) of valuations, bounds on counterfactual revenues.

RET in multi-unit auctions

- Revenue equivalence does not hold even under symmetric private valuations.
- Cannot be ranked for efficiency or revenue (Ausubel and Cramton, 2002).

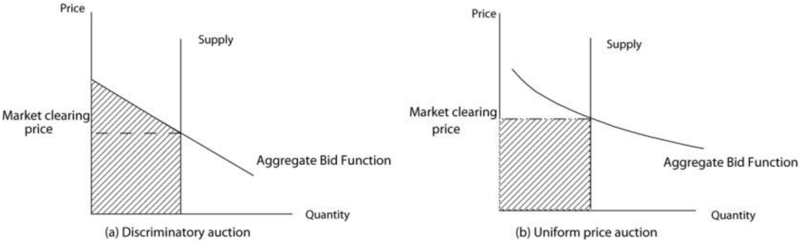


Fig. 1.—Discriminatory and uniform price auctions

Set-up

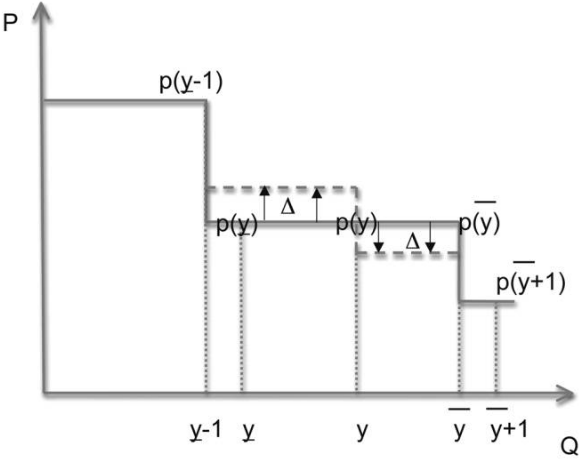
- Consider case of multiple homogeneous goods being sold simultaneously.
- Symmetric and risk-neutral bidders with IPVs.
- Valuations $v_i = \{v_i(1), v_i(2), ..., v_i(y^{\max})\}$.
- Offers $p_i = \{p_i(1), p_i(2), ..., p_i(y^{\max})\}$ (weakly decreasing).
- Equilibrium quantity $y_i(p)$ depends on all bids.
- Market clearing $Q = \sum_i y_i(p, v_i)$, also $y_i(p) = Q - \sum_{j \neq i} y_j(p, v_j)$.
- Profit takes expectation over y_i:

$$\Pi_i(p(\cdot), v_i(\cdot)) = \sum_y G(y; p(y))[v_i(y) - p(y)]$$

Bounds

- With non-increasing step bids, not all deviations are possible.
- Cannot lower a step in the middle, need to lower all the rest.
- Consider deviations where a "chunk" of the step is removed and monotonicity preserved.
- Increasing step gives upper bound, lowering step gives lower bound.
- *Note*: if willing to assume same valuation throughout the step or parametric functional forms on valuations, then one can recover point identification.

Deviations in the presence of steps



Deviations

- Consider a change in price Δ.
- Increasing step at y:

$$v(y) \leq \overline{v} \equiv p(y) + \Delta + \frac{\sum_{q=\underline{y}}^{y} G(q, p(y))}{\sum_{q=\underline{y}}^{y} [G(q, p(y) + \Delta) - G(q, p(y))]}.$$

- Decreasing step at y:

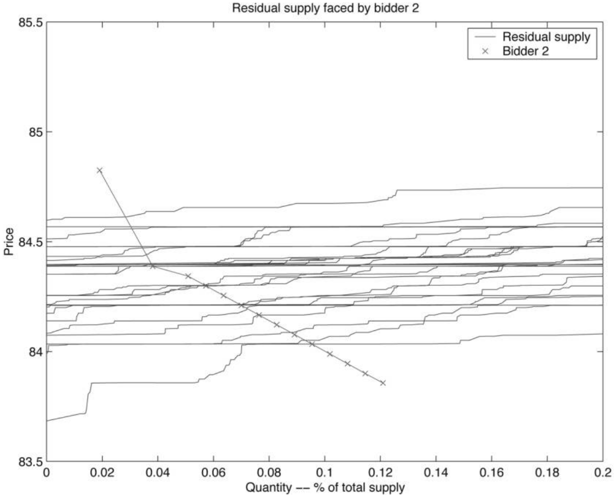
$$v(y) \leq \underline{v} \equiv p(y) + \frac{\sum_{q=y}^{\overline{y}} G(q, p(y) - \Delta)}{\sum_{q=y}^{\overline{y}} [G(q, p(y)) - G(q, p(y) - \Delta)]}.$$

- Encapsulates limiting case of continuous bids taking form similar to GPV (for multiple units).

Estimation

- As in GPV, need to approximate probability of winning different quantities at different price-quantity offers.
- Under strong assumption, can identify it using data from a single auction.
- Alternatively, pool across auctions that are comparable.
- Steps:
  1. Fix bidder i to identify $v_i(\cdot)$ (vector).
  2. Draw a random sample of $N-1$ bid vectors for other players (symmetry, independence).
  3. Construct realized residual supply for given strategies.
  4. With many draws, compute residual supply *distribution* and implied winning probabilities.

Resampling

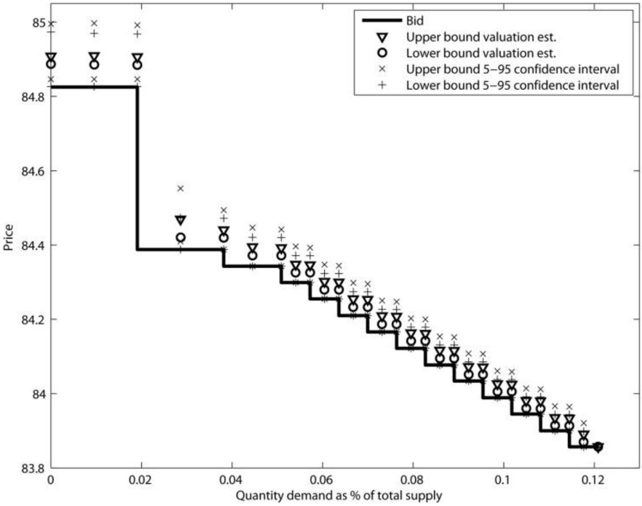


Residual supply faced by bidder 2
Residual supply
Bidder 2
Price
Quantity -- % of total supply
85.5
85
84.5
84
83.5
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

Bounds

- With distribution of residual supply, can estimate bounds using $\hat{G}$.
- There is a $\hat{G}$ and $\hat{G}_{\Delta}$ for each possible y of each bidder i.
- If y-grid is very fine, can take a sample of those.
- Some practical issues:
  - Bounds might not be identified if evaluated in a region outside from observed equilibrium prices (e.g., very high or very low bids).
  - Estimates can be sensitive to Δ, in practice it needs to be "large" enough to impact winning probabilities.
  - Additional smoothing approaches might be necessary.

Bounds



Bid
Upper bound valuation est.
Lower bound valuation est.
Upper bound 5−95 confidence interval
Lower bound 5−95 confidence interval
Price
Quantity demand as % of total supply
85
84.8
84.6
84.4
84.2
84
83.8
0
0.02
0.04
0.06
0.08
0.1
0.12

Mechanism Comparisons

- Estimation based on outcomes from discriminatory auction.
- Compare it to alternative settings: uniform and Vickrey auctions.
- Uniform: difficult to compute equilibrium, consider extreme case of truthful bidding as a bound.
- Upper bound on revenue to uniform and Vickrey auctions.
- Evaluate it at lower and upper valuation estimates.
- Simulate different auctions to use a sample of auctions consistent with approach.
- Cannot reject a zero difference between the two.

Mechanism Comparison Results

TABLE 3

Ex Ante Revenue and Efficiency Gains from Switching to a Uniform Price Auction with Truthful Bidding

Upper-Bound Revenue Gain (%)	Lower-Bound Revenue Gain (%)	Efficiency Gain (%)
.12 [−.07, .23]	.02 [−.11, .11]	.02 [.002, .035]

Note.—Numbers in brackets are 5–95 percent confidence intervals.

Related work

- Kastl (2011a): formalizes setting for both price and quantity deviations, application to Czech Treasury.
- Wolak (2003): applications to electricity auctions (uniform).
- Reguant (2014): application to electricity auctions in the presence of dynamic complementarities.

Problem set

- I will send problem set out after the holiday break.
- Cover foundations (LOV, GPV).
- Require some extensions.