## Production function estimation Empirical IO problem set

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The industry has the following "structural value-added" production function:

$$Q_{it} = \exp\left(\omega_{it} + \eta_{it}\right) \min\left\{L_{it}^{\beta_L} K_{it}^{\beta_K}, \beta_M M_{it}\right\}$$

where *i* indexes plants, *t* indexes years, *Q* is output, *L* is labor inputs, *K* is capital inputs, and *M* is materials inputs. The error term  $\eta_{it}$  is classical measurement error, and  $\omega_{it}$  is a serially correlated productivity residual.

The productivity term evolves according to an AR1 process:

$$\omega_{it} = \alpha_0 + \alpha_1 \omega_{i,t-1} + \varepsilon_{it}^{\omega}$$

where  $\varepsilon_{it}^{\omega}$  is i.i.d. across plants and time periods.

Input and output prices are exogenous from the perspective of an individual plant. There is idiosyncratic plant-level variation in wages:

$$p_{L,i,t} = p_{Lt} + \varepsilon_{it}^L$$

where  $\varepsilon_{it}^L$  is i.i.d. across plants and time periods. All plants face the same materials prices and rental rates.

Capital is a dynamic input following a standard model of accumulation:

$$k_{i,t+1} = .9k_{it} + i_{it}.$$

Labor and materials are static inputs.  $L_{it}$  and  $M_{it}$  are set simultaneously after observing  $\omega_{it}$ . The variable inputs are set to maximize profits in the current period.

To keep things relatively simple, the simulated data set has no entry or exit, and input use and investment levels are always strictly positive.

1. Describe an empirical approach for estimating  $\beta$  which is appropriate given the theoretical setup above. Describe how the approach differs from Olley-Pakes and/or Levinsohn-Petrin, and state what your strategy requires in terms of assumptions monotonicity of input demands or investment.

**2.** Using the attached data, estimate the coefficients of the production function  $(\beta)$  using OLS, fixed effects, and your control function approach. Explain the differences in the estimates of the  $\beta$ 's.

**3.** Compute the average markup (ratio of output price to marginal cost). Explain how this is done.

**4.** Is there any issue with the identification of  $\beta_l$  or  $\beta_m$  here? Why or why not? What if  $\varepsilon_{it}^L$  were serially correlated?

Please turn in your solutions via email to ptscott@gmail.com.